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REPRESENTATION OF THE STATOR TO ROTOR SELF- AND MUTUAL INDUCTANCES IN A SALIENT POLE SYNCHRONOUS GENERATOR IN THE NO-LOAD STATE

Abstract: This paper presents a representation of the stator to rotor self and mutual inductance and derivative distributions in the 5.5 kVA salient pole synchronous generator with damping circuits, both with and without a rotor skew in the no-load steady state. In the circuital linear and nonlinear model, the stator to rotor mutual inductances are determined in the FEM program. Presented distributions allow to express the waveforms of induced phase stator voltages in the no-load steady state using a circuital model in the natural reference frame of the stator and rotor.

1. Introduction

A 3-phase synchronous generator is used as a constantly increasing reserved power source. A synchronous generator as a source of power has good quality if the contents of the higher harmonic induced in armature windings are very low [1, 2]. The content of higher harmonic voltages induced in the armature windings is particularly evident in low power up to several kVA salient pole synchronous generators especially [3]. The largest content of harmonics in the induced phase voltage (in low power generators) occurs with the synchronous generators with the single-layer stator winding without the rotor (or stator) skew. In 3-phase low power synchronous generators (less than 10 kVA) dominate single layer windings. Apart from that, many low-cost synchronous generators in the Polish market have the stator and rotor structures without a skew. No skew causes a significant increase in higher harmonic order in stator and rotor inductance distributions [3, 4] in the induced voltages [1, 2, 4] and in currents during powering of various types of electronic equipment (AV, etc.), computers, notebooks, UPS, compact fluorescent lamps, etc. The presence of higher harmonics in the phase voltage and current waveforms of synchronous generators has negative economic consequences. Because of additional power losses these generator sets consume more fuel.

In dynamic states the electromagnetic properties of a synchronous machine depend on the presence of the damping circuits [5, 6]. There are many methods to obtain the parameters of the damping circuits from experimental, analytical or FEM methods [5, 6]. The damping circuits allow for shortening many transients' stages, e.g. the hunting [6], reducing higher harmonics in the field winding current or increasing of higher harmonics in armature currents [7]. As will be shown in the no-load steady state, the discrete distribution of the damping bars due to higher harmonic current has an influence on higher harmonic contents in induced voltages.

In the no-load of the steady state of a salient pole synchronous generator the distribution of magnetic flux density in the air gap is distorted due to: saturation of the magnetic circuit (mainly low order odd harmonics) [8, 9], the influence of the stator slot opening or rotor damping cage slot opening [10], magnetic rotor asymmetry [5, 9, 11, 12] and static and dynamic stator and rotor eccentricity [9, 11, 12].

In circuital modelling an analytical way of describing a distribution of the self- and mutual inductances with taking into account the electrical angle of the rotor position and current are described in [9, 10, 11, 13]. However, this method requires correction of coefficients describing the self- and mutual inductances [14, 15]. These coefficients the most frequently are expressed in the form of Fourier series or based on the co-energy are determined as a function of the electrical angle of the rotor position and current using analytical or FEM methods [12 - 16]. The correction of the analytical form of coefficients the most often is carried out on the basis of experimental investigation or using the FEM methods (if the detailed geometry and construction-material data are known) [14].

This article examines the influence of representation of the stator to the rotor winding mutual inductances on the induced voltages in the 3-phase armature winding in a 5.5 kVA, salient pole synchronous generator both with and without rotor skew under no-load steady state conditions. In part 2 of this article, the higher harmonic contents in induced stator phase voltages are calculated when the DC voltage or DC current is powering the field winding. The symmetry of the stator windings and field winding with 10 equivalent shorted damping bars, with the linearity and nonlinearity of the magnetic circuit have been assumed. The self- and mutual inductances are determined using Flux 2D Skew and FEMM programs [3, 4]. The end effects (leakage inductance and resistance) are calculated using the analytical technique.

2. Model of a salient pole synchronous generator in the stator and rotor natural reference frame

The Park's transformation of the higher harmonics of the stator self- and mutual inductance distributions to the dq0-axes [9, 14, 17] does nothing and only introduces additional unnecessary calculations. In this case the self- and mutual inductance distributions in the dq0-axes are dependent on the angle position of the rotor. Hence, the induced phase stator voltages u_a, u_b and u_c in simulations is easier to carry out with a circuital model of a salient pole synchronous generator in the stator and rotor natural reference frame.

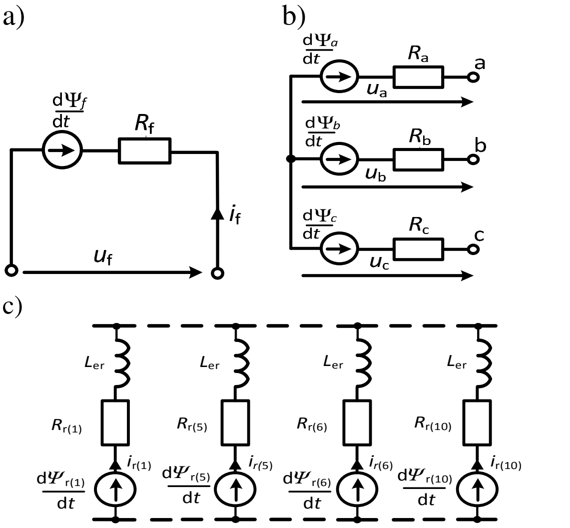

Fig. 1. Equivalent circuit parameters of a salient pole synchronous generator in the stator and rotor natural reference frame in the no-load state a) field winding, b) stator windings, c) damping circuits

Figure 1 shows the equivalent circuit parameters of a synchronous generator in the stator and the rotor natural reference frame in the no-load state (i_a, i_b and i_c are equal to zero). The equivalent circuit represents the stator windings, the field winding and shorted equivalent 5-damping bars per pole.

For such arranged equivalent circuit (Fig. 1), the voltage u_a, u_b, u_c of a salient pole synchronous generator induced in the three-phase armature windings, taking into account the field winding, shorted damping bars and the electrical angle of the rotor position can be derived from the equations, in stator coordinates (for the stator windings) and in rotor coordinates (for the rotor windings).

$$\frac{d\psi_a}{dt} = u_a, \quad \frac{d\psi_b}{dt} = u_b, \quad \frac{d\psi_c}{dt} = u_c \tag{1}$$

$$\frac{d\psi_f}{dt} + R_f i_f = u_f \tag{2}$$

$$\frac{d\psi_{r(k)}}{dt} + R_{r(k)} i_{r(k)} + L_{er}\frac{i_{r(k)}}{dt} = 0 \tag{3}$$

$$\frac{d\theta}{dt} = \omega \quad \text{and} \quad i_{r(1)} + i_{r(2)} + \ldots + i_{r(10)} = 0 \tag{4}$$

Where: a, b and c – indexes of stator windings, f – field winding index, $r(k)$ – index of kth-damping bar, $k = \{1, 2, \ldots 10\}$, Ψ_a, Ψ_b, Ψ_c – stator linkage fluxes, u_a, u_b, u_c – stator phase voltages, θ – electrical angle of the rotor position, $\theta = \theta_m p_b$, θ_m – mechanical angle of the rotor position, $\omega = (1/p_b)\cdot d\theta/dt$ – electrical angular velocity, R_f – resistance of the field winding, i_f – field current, $i_{r(1)}, \ldots, i_{r(10)}$ – current in equivalent 10 damping bars and ring elements, $R_{r(1)}, \ldots, R_{r(10)}$ – resistance of the equivalent 10-damping bars and ring elements, respectively, whereas, $R_{r(k)}$ – resistance of equivalent damping bar and ring elements $R_{r(k)} = R_{pr} + R_{er}/\{2\sin(\alpha_{(k)}/Q_r)\}$, R_{pr}, R_{er} – resistance of damping bar and ring elements, respectively, $\alpha_{(k)}$ – is the angle between the equivalent k – rotor damping bar (with ring elements) and the rotor reference axis, Q_r – number of rotor bars, L_{er} – inductance of end ring elements.

The equation for differential linkage fluxes (1) - (3) taking into account the influence of currents and electrical angle of the rotor position θ can be derived from the equations [9,13, 18]:

$$\frac{d\boldsymbol{\Psi}}{dt} = \frac{d\theta}{dt}\frac{\partial \mathbf{L}(\mathbf{i},\theta)}{\partial \theta}\mathbf{i} + \mathbf{L}(\mathbf{i},\theta)\frac{d\mathbf{i}}{dt} \tag{5}$$

Based on the expressions (1) - (3) and (5) and the mutual inductances of the stator to rotor windings and the self inductance of the field winding, the induced phase stator voltages u_a, u_b, u_c can be expressed as:

$$\mathbf{u}_s = \omega\frac{\partial \mathbf{L}_{sr}}{\partial \theta}\mathbf{i}_r + \mathbf{L}_{sr}\frac{d\mathbf{i}_r}{dt} \tag{6}$$

However, the equation describing the excitation of the field winding and damping bars can be expressed as

$$\mathbf{u}_{fr} = \left(\omega\frac{\partial \mathbf{L}_{fr}}{\partial \Theta} + \mathbf{R}_{fr}\right)\mathbf{i}_{fr} + \mathbf{L}_{fr}\frac{d\mathbf{i}_{fr}}{dt} \tag{7}$$

Where: $\mathbf{u}_s = [u_a, u_b, u_c]^T$ – matrix of induced stator phase voltages, $\mathbf{L}_{sr} = \begin{bmatrix} L_{af} & L_{ar(1)} & \cdots & L_{ar(10)} \\ L_{bf} & L_{br(1)} & \cdots & L_{br(10)} \\ L_{cf} & L_{cr(1)} & \cdots & L_{cr(10)} \end{bmatrix}$ – matrix of mutual inductances of stator-to-rotor damping bars and ring elements, L_{af}, L_{bf}, L_{cf} – mutual inductance of stator-to-field windings, $L_{ar(1)}$, $L_{br(1)}$, $L_{cr(1)}$, … $L_{ar(10)}$, $L_{br(10)}$, $L_{cr(10)}$ – mutual inductance of stator windings to 10 damping bars (and ring elements), $\mathbf{L}_{fr} = \begin{bmatrix} L_f + L_{ef} & L_{fr(1)} & \cdots & L_{fr(10)} \\ L_{fr(1)} & L_{r(1)} + L_{er} & \cdots & L_{r(1,10)} \\ \cdots & \cdots & & \cdots \\ L_{fr(10)} & L_{r(10,1)} & \cdots & L_{r(10)} + L_{er} \end{bmatrix}$ – matrix of self- and mutual inductances of field-to-damping bars and ring elements, L_f – self inductance of the field winding, $L_{fr(1)}$, … $L_{fr(10)}$ – mutual inductance of field winding to 10 damping bars and ring elements, $L_{r(1)}$, … $L_{r(10)}$ – self inductance of 10 damping bars and ring elements, $\mathbf{i}_r = [i_{r(1)}, \ldots, i_{r(10)}]^T$ – matrix currents in damping bars, $\mathbf{i}_{fr} = [i_f, i_{r(1)}, \ldots, i_{r(10)}]^T$ – matrix of field winding current and currents in damping bars, $\mathbf{u}_{fr} = [u_f, 0, \ldots, 0]^T$ – matrix of voltages of field winding and in shorted 10 damping bars and ring elements, respectively, $\mathbf{R}_{fr} = [R_f, R_{r(1)}, \ldots, R_{r(10)}]$ – diagonal matrix of resistance of the field winding and 10 damping bars and ring elements, $L_{r(m,n)}$ – mutual inductance of damping bars and ring elements (m ≠ n) and $m, n = \{1, \ldots, 10\}$ [18].

3. Determination of the self- and mutual inductances

The self- and mutual inductances in the expressions (6) - (7) are calculated on the basis of the real construction data (non-uniform air gap at the periphery of the stator and rotor) received from the manufacturer of synchronous generators that are used frequently in generator sets in Poland. While calculating the inductance distributions (as a function of the electrical rotor position angle θ, with 10 damping bars and nonlinearity) in the Flux 2D Skew and FEMM software [3, 4], the factory single-layer winding placed in the stator slots is taken into account. The self- and mutual inductances are carried out for a salient pole synchronous generator (with and without the rotor skew) rated: S_N = 5.5 kVA, U_N = 400 V (Y), n_N = 3000 rpm, I_N = 7.9 A, $\cos\varphi_N$ = 0.8, Q_s = 24 (number of stator slots), α_q = 15° (factory rotor skew equal to stator slot pitch), p_b = 1 (number of pole pairs).

Figure 2 presents magnetic flux distribution lines of the examined 5.5 KVA nonlinear salient pole synchronous generator in the no load steady state with shorted equivalent damping 5 bars per pole with initial rotor position θ_0 = 0. A method of determining the self- and mutual inductance distributions in FEMM program is detailed presented in [18].

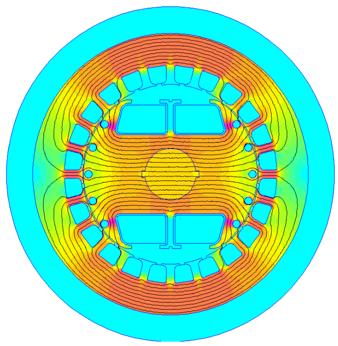

Fig. 2. Magnetic flux distribution lines in the 5.5-kVA salient pole synchronous generator

Figure 3 shows the comparison of the distributed stator to field winding mutual inductances and their derivatives for linear and nonlinear 5.5 kVA salient pole synchronous generator with and without the rotor skew. The mutual inductances for nonlinear model are very similar to the linear ones. The differences are only visible after calculation of the product of $\omega\partial L_{afn}/\partial\theta$ and due to Fourier analysis.

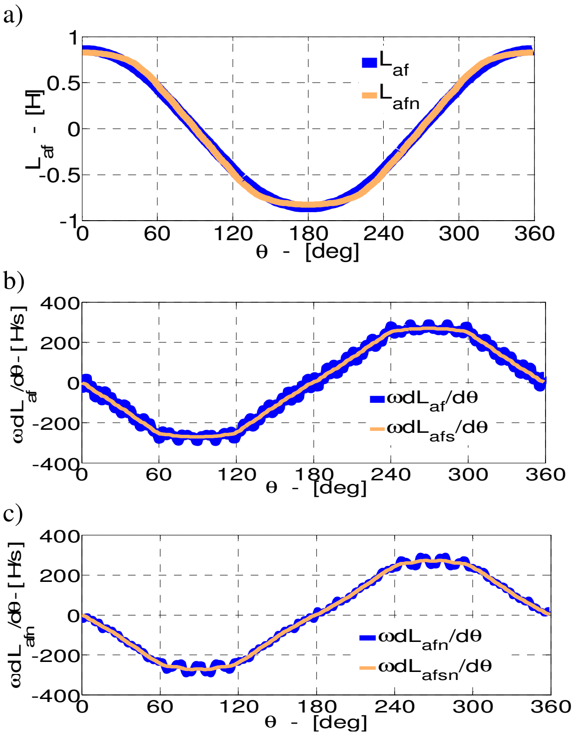

Fig. 3. Comparison of the stator to field winding mutual distributions of a) inductances and b), c) the product of $\omega\partial L_{af}/\partial\Theta$ for linear and nonlinear model with and without the rotor skew, respectively

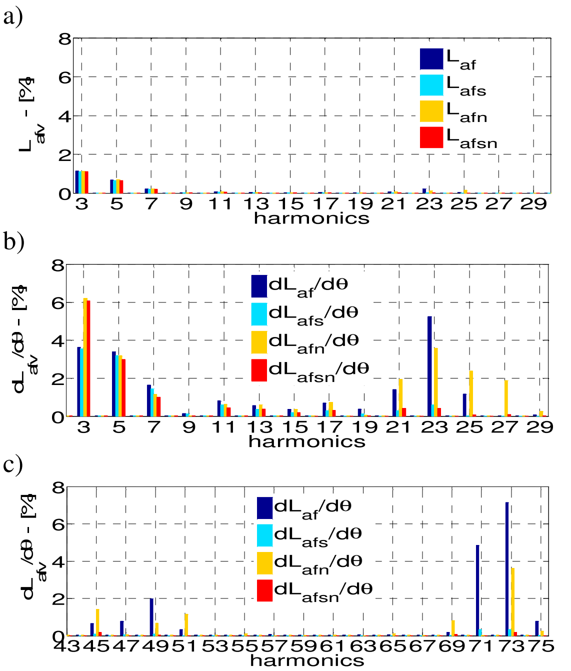

Fig. 4. Comparison of harmonic contents for linear and nonlinear model of the stator (phase a) to field winding mutual distributions of a) inductance from 3rd to the 29th ones, b), c) derivative from 3rd to the 29th and 43rd to the 75th ones, respectively

Figure 3 indicates: s – index with the rotor skew, n – index for nonlinear model. Detailed analysis of the self- and mutual inductance distributions for the salient pole synchronous generator is described in [3, 4].

Figure 4 shows the contents of harmonic magnitudes due to Fourier analysis in inductances L_{af} and derivatives $\partial L_{af}/\partial\theta$ for linear and nonlinear model (Figs. 3a - 3c) with and without the rotor skew.

The contents of harmonic in inductances (Fig. 4) from 43rd to the 75th harmonic are less than 0.05%. Figure 4 shows that the greatest reduction of the harmonic content is achieved by using the rotor skew (or stator skew).

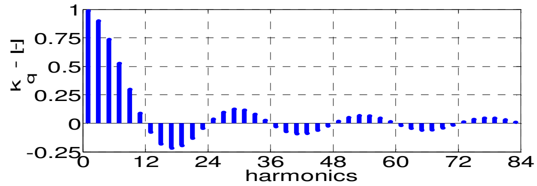

Fig. 5. Skew factor k_{qv} for the α_q = 15° and p_b = 1

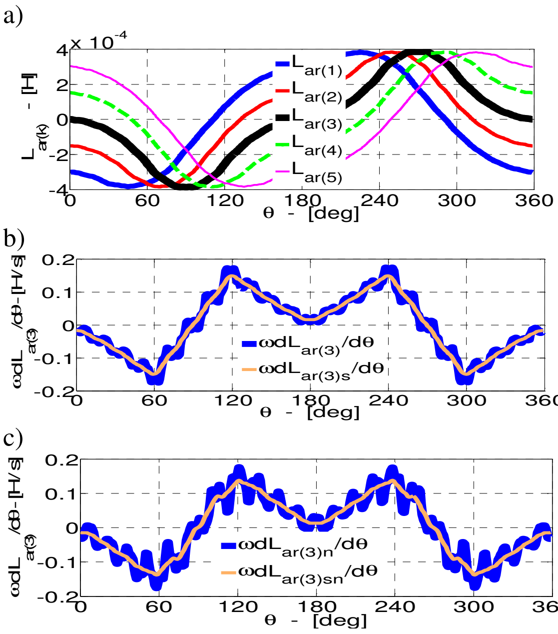

Fig. 6. Comparison of the stator to damping bar mutual distributions of a) inductances for damping bars $L_{ar(1)}$ - $L_{ar(5)}$ ($L_{ar(6)}$ - $L_{ar(10)}$), b), c) the product of $\omega\partial L_{ar(3)}/\partial\theta$ for linear and $\omega\partial L_{ar(3)n}/\partial\theta$ for nonlinear model with and without the rotor skew, respectively

Figure 5 shows the values of the skewing factor $k_{q(\nu)}$ for the ν-th harmonic of the examined synchronous generator (α_q = 15° - equal to the stator slot pitch and p_b = 1). Figures 4 and 5 show that the skew of the rotor (or stator)

causes a significant reduction in higher harmonic order. Lower-order harmonics caused by e.g. the saturation of the magnetic circuit due to the rotor skew are only slightly reduced.
Figure 6 shows the comparison of the stator to damping bars $L_{ar(1)}$ - $L_{ar(5)}$ ($L_{ar(6)}$ - $L_{ar(10)}$) distribution of mutual inductances and the product of $\omega\partial L_{ar(3)}/\partial\theta$ for the linear- and $\omega\partial L_{ar(3)n}/\partial\theta$ for the nonlinear salient pole synchronous generator.
The distributed stator to damping bar mutual inductances for nonlinear model are very similar to the linear ones and therefore not shown in Figure 6. The differences are only visible after the calculation of the derivatives, the comparison of which, due to Fourier analysis (in relation to the fundamental component of $L_{ar(3)}$), is shown in Figure 7.

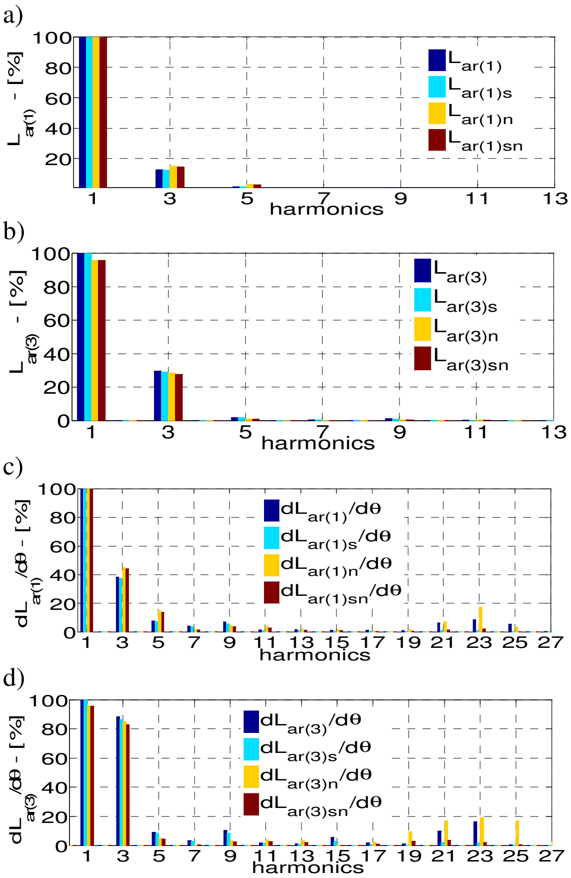

Fig. 7. Harmonic contents of the stator to damping bars of a), b) mutual inductances $L_{ar(1)}$ and $L_{ar(3)}$, c), d) derivatives $\partial L_{ar(1)}/\partial\theta$ and $\partial L_{ar(3)}/\partial\theta$ for linear and nonlinear model with and without the rotor skew

From expressions (6) and (7) results that u_a, u_b and u_c depend on the field current and currents in damping bars. The field current (7) depends on the method of powering the field winding [3] and on the self- and mutual inductances $\mathbf{L}_{fD}$.

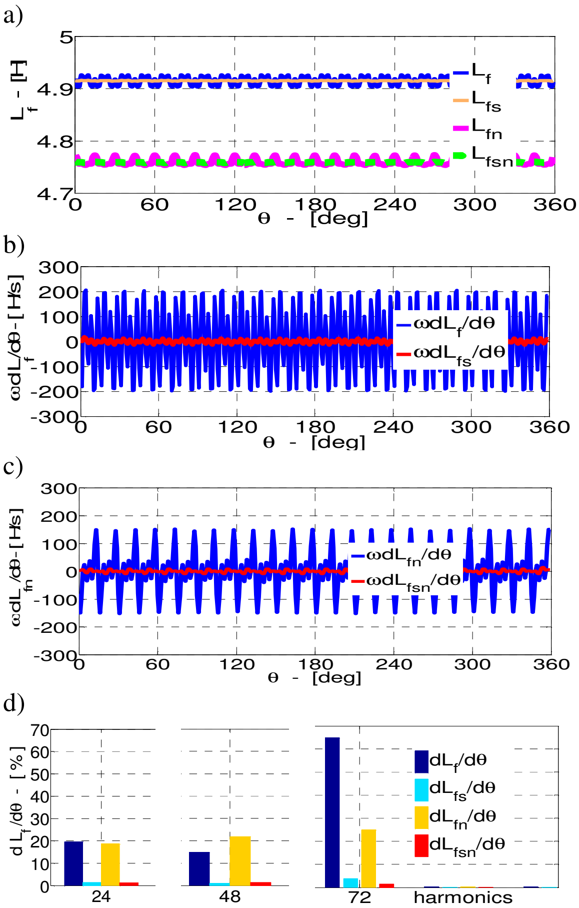

Fig. 8. Comparison of the a) field winding self inductance distributions for linear and nonlinear model with and without the rotor skew, b), c) products of $\omega(\partial L_{fs}/\partial\theta)$, $\omega(\partial L_f/\partial\theta)$ and $\omega(\partial L_{fsn}/\partial\theta)$, $\omega(\partial L_{fn}/\partial\theta)$, d) detailed harmonic contents of self inductance derivatives for 24th, 48th and 72nd order

Figure 8 shows the distributions of the field winding self inductance, the product of $\omega(\partial L_f/\partial\theta)$ and their harmonic contents in relation to the constant component of L_f.

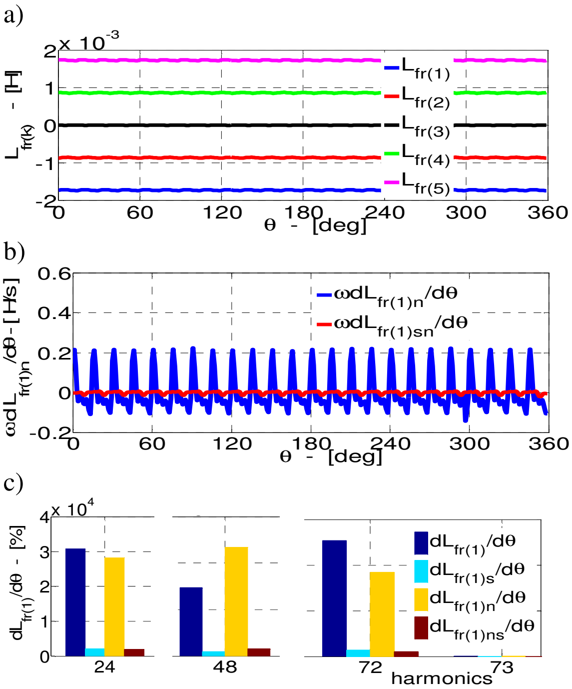

Fig. 9. Distributions of a) the field winding to damping bars mutual inductance, b) the products $\omega(\partial L_{fr(1)sn}/\partial\theta)$, $\omega(\partial L_{fr(1)n}/\partial\theta)$, c) harmonic contents of the mutual inductance derivatives with and without the rotor skew

Figure 9 shows the comparison of the mutual inductance distributions of field winding to damping bars $L_{fr(1)n}$ - $L_{fr(5)n}$ ($L_{fr(6)n}$ - $L_{fr(10)n}$), the product of $\omega(\partial L_{fr(1)sn}/\partial\theta)$ and $\omega(\partial L_{fr(1)n}/\partial\theta)$ for nonlinear model (with and without the rotor skew) and the contents of harmonic magnitudes (in relation to the constant component of $L_{fr(1)}$). The differences between the linear and nonlinear mutual inductance distributions are very small and therefore are only shown for nonlinear model in Figure 9. Moreover, the other components of the products $\omega(\partial L_{fr(k)s}/\partial\theta)$, $\omega(\partial L_{fr(k)}/\partial\theta)$ and $\omega(\partial L_{fr(k)sn}/\partial\theta)$, $\omega(\partial L_{fr(k)n}/\partial\theta)$ for bars $k = \{2, \ldots, 5\}$ are shifted by $2\pi/k$ in electrical degree and are very similar to the mutual inductance distribution of field winding to damping bars $\omega(\partial L_{fr(1)}/\partial\theta)$.

4. Conclusion

Comparing linear and nonlinear model of the 5.5-kVA salient pole synchronous generator with and without the rotor skew with 10 damping bars, it can be concluded that the magnetic saturation reduces the harmonic contents in voltages of:

- $\omega(\partial L_{afn}/\partial\theta)i_{fn}$, $\omega(\partial L_{bfn}/\partial\theta)i_{fn}$, $\omega(\partial L_{cfn}/\partial\theta)i_{fn}$ (product of ω and derivatives of mutual inductances of stator to field winding and field winding current),
- $\omega(\partial L_{fn}/\partial\theta)i_{fn}$, (product of ω and derivatives of self inductances of field winding and field winding current).

Magnetic saturation has a very small influence on changing harmonic contents in derivatives of mutual inductance of field winding to damping bars $\partial L_{fr(1)n}/\partial\theta$ - $\partial L_{fr(5)n}/\partial\theta$ ($\partial L_{fr(6)n}/\partial\theta$ - $\partial L_{fr(10)n}/\partial\theta$).

The greatest reduction of the harmonic content in the self- and mutual inductance distributions is achieved by using the skew of the rotor.

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